JWFront

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JWFront is a program written in Java to visualize, in two or three dimensions, the wavefronts passing by an object, i.e. a black hole or gravitational lens. The light cones can also be displayed. The caustics for a given metric can be visualized in two or three dimensions. This program has a graphical user interface to control input and output. The implemented metrics so far are: Minkowski, Schwarzschild, and Kerr. We will describe how the program works and present its applications.

1. Wavefronts and Caustics in General Relativity

The mathematical theory of wavefronts and caustics in general relativity were developed by Friedrich and Stewart[1] (see also Stewart[2]). They used Arnold's singularity theory[3] to tackle this problem. More recently Hasse[4] *et al.*, Low,[5] and Ehlers and Newman[6] have revived this topic. Petters[7] (see also Petters[8] *et al.*), and Frittelli and Petters[9] developed this theory for gravitational lenses. More recently, Grave[10] made movies in MPG format of the gravitational collapse and wavefronts in presence of Schwarzschild and Kerr fields. To have a perspective on this topic, the interested reader is referred to Perlick's paper.[11]

1.1. *Wavefronts*

A wavefront is generated by a bundle of light rays orthogonal to a spacelike 2-surface in a four-dimensional Lorentzian manifold.[4] For the sake of visualization, the wavefront is defined as the surface $\mathcal{A}$ generated by all points of the null geodesic bundle at a given time t_i:

$$\mathcal{A}(t_i) = \{\gamma(t_i) \mid \gamma(t_i) \text{ is a null geodesic with } \gamma(t_0) = (t_0,\, x_0,\, y_0,\, z_0),\; t_i \geq t_0\}\,.$$

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To get the wavefronts, the geodesic equation is solved for a bundle of light rays starting from a common point.

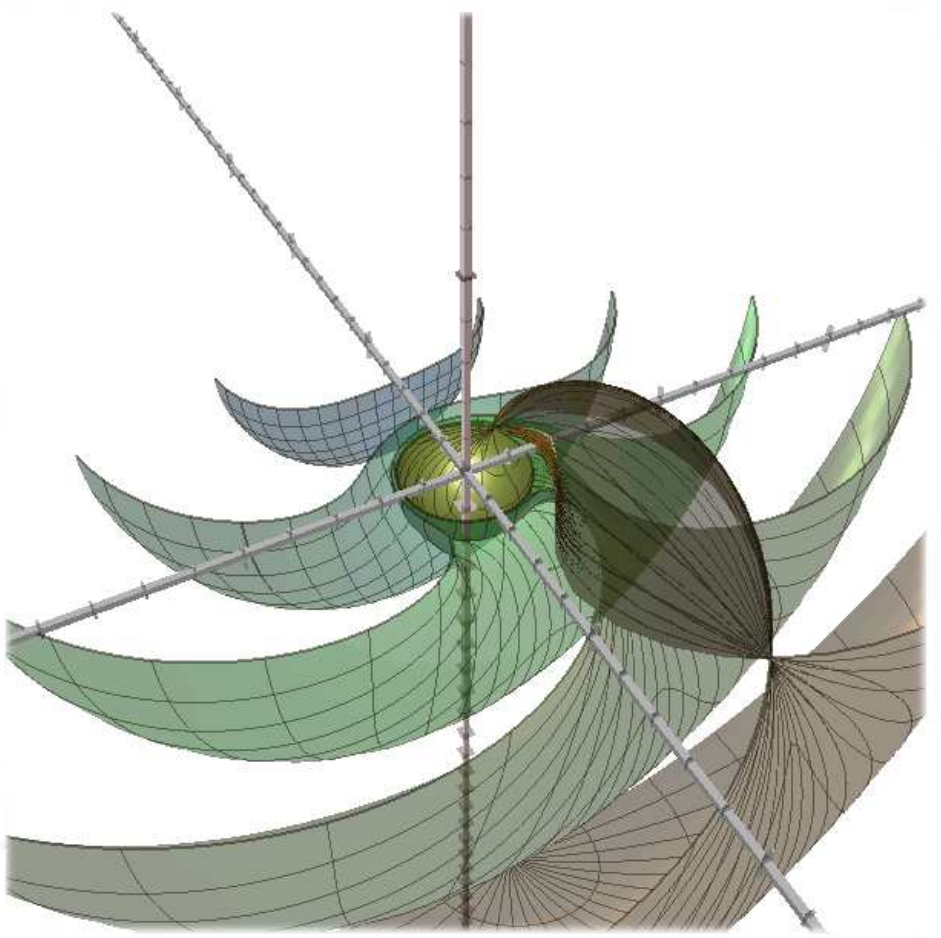

Fig. 1. A 3d visualization of the wavefront for the Kerr metric. The structure of the caustics can be seen.

1.2. *Caustics*

The caustic of a wavefront is defined as the set of all points where the wavefront fails to be an (immersed) submanifold.[4] Roughly speaking, a caustic appears if the wavefront crosses itself.

1.3. *Light Cones*

The light cone is defined as the surface generated by all points (x, y, t) that fulfil the geodesic equation with the null geodesic condition, where the axes are x, y, and t (t as the z axis). Light cones can be used to visualize caustics.

2. JWFront

JWFront, our program interface, was written in Java and uses LightWeight Java OpenGL Library (lwjgl), which native drivers support Windows, Linux and Mac computers. With this graphical user interface, the user gets, after entering the initial data, his or her visualizations practically in real time depending on the speed of the computer. The program can be downloaded from the following Website:

`http://www.tat.physik.uni-tuebingen.de/~boot/`

3. Applications

Among the applications of the program are:

- wavefront animations in 2d and 3d
- light cone structure visualizations

The visualization of a 3d wavefront for the Kerr metric is shown in Fig. 1. In this picture, one sees that the deformation of the wavefront occurs not only in 2d but in 3d as well. For Schwarzschild, such distortion is also possible. To see more examples visit our website:

`http://www.tat.physik.uni-tuebingen.de/~boot/jwfront-doc/`

4. Conclusions

JWFront is a very useful interactive program to visualize wavefronts and the light cone structure for the most important metrics in general relativity and astrophysics. The applications presented here show that it helps understand the light propagation in strong fields, such as in Schwarzschild and Kerr spacetimes. The caustics produced by the chosen metric can be easily visualized in two or three dimensions, and also with the light cone.

References

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